Arithmetic, Logical and Matrix operations in R

In this lecture

- Arithmetic
- Logical
- Matrix operations

Arithmetic operations in R

Arithmetic operations in R

Symbols	Operation
=, < –*	*Assignment*
+	*Addition*
-	*Subtraction*
*	*Multiplication*
/	*Division*
^,**	*Exponent*
%%	*Remainder*
%/%	*Integer division*

* *In R only '< –' is valid for assignment operation where as in R Studio both = and < –' will work*

Hierarchy of operations

$$A = 7 - 2\,x\,\frac{27}{3^2} + 4$$

Order of Precedence	Operation
Bracket	( )
Exponent	^,**
Division	/
Multiplication	*
Addition and subtraction	+,-

Logical operations in R

Logical operations in R

Symbols	Operation	Examples
<	*Less than*	
<=	*Less than equal to*	
>	*Greater than*	
>=	*Greater than equal to*	
==	*Exactly equal to*	
!=	*Not equal to*	
!	*Not*	
|	*Or*	
&	*And*	
isTRUE	*Test if variable is TRUE*	

```
> 2>3
[1] FALSE
>
> 2<3
[1] TRUE
>
> 2>=3
[1] FALSE
>
> 2<=3
[1] TRUE
>
> 2==3
[1] FALSE
>
> !2
[1] FALSE
>
> 2!=3
[1] TRUE
>
> |
```

Matrix operations in R

Matrices

A matrix is a rectangular arrangement of numbers in rows and columns

Rows run horizontally and columns run vertically

$$\begin{pmatrix} 1 & 5 & 3 \\ 4 & 9 & 2 \\ 5 & 6 & 7 \end{pmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$$

Creating matrices

Follow these steps to create a matrix

1. Open a curve bracket,

A = matrix()

2. Enter the sequence of elements,

A= matrix(c(1,2,3,4,5,6,7,8,9))

3. Specify the parameters nrow, ncol, byrow

A= matrix(c(1,2,3,4,5,6,7,8,9), nrow =3, ncol=3, byrow=TRUE)

Console ~/

```
> A= matrix(c(1,2,3,4,5,6,7,8,9),
+           nrow =3,
+           ncol=3,
+           byrow=TRUE)
> A
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
```

This parameter decides how values in the vector would be assigned i.e. “by row” or not

Creating special matrices

Different ways of creating matrices:

a. **Matrix where all rows and columns are filled by a single constant 'k'.**
   - ➢ **For k=3, with 'm' rows & 'n' columns**

     Command :matrix(3,m,n)

b. **Diagonal matrix:**
   - ➢ **Values in diagonal, similar to 'matrix()'.**
   - ➢ **Mention 'k' as constant/array in first parameter.**
   - ➢ **Command: diag(k,m,n)**

c. **Identity matrix:**
   - ➢ **Use 'diag()' command with k=1**

Console ~/

```
> matrix(3,3,4)
     [,1] [,2] [,3] [,4]
[1,]    3    3    3    3
[2,]    3    3    3    3
[3,]    3    3    3    3
> diag(c(4,5,6),3,3)
     [,1] [,2] [,3]
[1,]    4    0    0
[2,]    0    5    0
[3,]    0    0    6
```

```
>diag(1,3,3)
[,1] [,2] [,3]
[1,] 1 0 0
[2,] 0 1 0
[3,] 0 0 1
```

Exercise: Creating matrices

Create the following matrices in R

$$\begin{bmatrix} 3 & 5 \\ -2 & 0 \end{bmatrix} \quad , \quad \begin{bmatrix} 1 & 10 \\ 3 & -1 \\ 7 & 5 \end{bmatrix}$$

and

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ -1 & -2 & -3 \\ 5 & 4 & 3 \end{bmatrix}$$

Matrix metrics

 # create a matrix A

A= matrix(c(1,2,3,4,5,6,7,8,9), nrow =3, ncol=3, byrow=TRUE)

Finding the size of the matrix, A :

dim(A) will return the size of the matrix

nrow(A) will return the number of rows

ncol(A) will return the number of columns

prod(dim(A)) or length(A) will return the number of elements

Console ~/

```
> dim(A)
[1] 3 3
> nrow(A)
[1] 3
> ncol(A)
[1] 3
> length(A)
[1] 9
> 
```

Accessing, editing, deleting in elements in matrices

They follow the same convention as dataframes such as

- Array/value before "," for accessing rows
- Array/value before "," for accessing columns
- use of '-' for removing rows/columns
- Strings can be assigned as names of rows and columns using:
    - rownames() and Colnames()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 9 & 1 \end{bmatrix}_{3\times3}$$

```
> A=matrix(c(1,2,3,4,5,6,8,9,1),
+          3,3,byrow = T)
>
> colnames(A) <- c("a","b","c")
> rownames(A) <- c("d","e","f")
> A
  a b c
d 1 2 3
e 4 5 6
f 8 9 1
> A[,1:2]
  a b
d 1 2
e 4 5
f 8 9
> A[,c("a","c")]
  a c
d 1 3
e 4 6
f 8 1
> A[c("d","f"),]
  a b c
d 1 2 3
f 8 9 1
> |
```

Accessing an entry of a matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

First row, second column

```
> A[1,2]
[1] 2
>
```

Second row, third column

```
> A[2,3]
[1] 6
>
```

The part before the comma should be the row number
The part after the comma should be the column number

Accessing a column

- Specify the column index
- Leave the rows index unspecified
- This means accessing all row elements of the given column index

$$A = \begin{bmatrix} \boxed{\begin{matrix} 1 \\ 4 \\ 7 \end{matrix}} & \begin{matrix} 2 \\ 5 \\ 8 \end{matrix} & \begin{matrix} 3 \\ 6 \\ 9 \end{matrix} \end{bmatrix}$$

All rows in first column

```
> A[,1]
[1] 1 4 7
>
```

Accessing a row

$$A = \begin{bmatrix} 1 & 2 & 3 \\ \boxed{4 \quad 5 \quad 6} \\ 7 & 8 & 9 \end{bmatrix}$$

```
> A[2,]
[1] 4 5 6
>
```

Leaving the column index empty means choose all the columns

How do you access the last row ?

A[nrow(A),]

Accessing everything but one column

$$A = \begin{bmatrix} 1 & \boxed{2} & 3 \\ 4 & \boxed{5} & 6 \\ 7 & \boxed{8} & 9 \end{bmatrix}$$

- Access the column that has to be avoided and then put a '-' sign in front of it
  - For example: A[,-2]
  - This will fetch all the columns except the 2nd column

```
> A[,-2]
     [,1] [,2]
[1,]    1    3
[2,]    4    6
[3,]    7    9
```

Accessing everything but one row

$$A = \begin{bmatrix} 1 & 2 & 3 \\ \boxed{4 \quad 5 \quad 6} \\ 7 & 8 & 9 \end{bmatrix}$$

```
> A[-2,]
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    7    8    9
>
```

- Access the row that has to be avoided and then put a '-' sign in front of it
  - For example: A[-2,]
  - This will fetch all the row except the 2nd row

Exercise: Accessing elements of a matrix

Do the following in R

Assign the following matrix

$$A = \begin{bmatrix} 1 & 7 & 3 \\ 4 & 4 & 6 \\ 4 & 7 & 12 \end{bmatrix}$$

- Change the element 12 to 13
- Access the second row and the third column
- List all the elements in the second column and third row

Colon operator

Colon operator can be used to create a row matrix

```
> 1:10
 [1]  1  2  3  4  5  6  7  8  9 10
>
> 10:1
 [1] 10  9  8  7  6  5  4  3  2  1
>
```

Colon operator: sub matrices selection

The colon notation can also be used to pick sub-matrices

Sub-matrix

$$A = \begin{bmatrix} \boxed{\begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}} & \begin{matrix} 3 \\ 6 \\ 9 \end{matrix} \end{bmatrix}_{3\times 3}$$

The sub-matrix occupies the **first three rows** and the **first two columns**

```
> A[1:3,1:2]
     [,1] [,2]
[1,]    1    2
[2,]    4    5
[3,]    7    8
>
```

```
> A[1:3,-3]
     [,1] [,2]
[1,]    1    2
[2,]    4    5
[3,]    7    8
>
```

```
> A[,1:2]
     [,1] [,2]
[1,]    1    2
[2,]    4    5
[3,]    7    8
>
```

Accessing submatrices: Example 2

$$A = \begin{bmatrix} \boxed{1 \quad 2} & 3 \\ 4 \quad 5 & 6 \\ \boxed{7 \quad 8} & 9 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 7 & 8 \end{bmatrix}$$

```
> A[c(1,3),1:2]
     [,1] [,2]
[1,]    1    2
[2,]    7    8
>
```

```
> A[c(1,3),c(1,2)]
     [,1] [,2]
[1,]    1    2
[2,]    7    8
>
```

Exercise: Accessing sub-matrices

$$A = \begin{bmatrix} \boxed{1} & 2 & \boxed{3} \\ \boxed{4} & 5 & \boxed{6} \\ 7 & 8 & 9 \end{bmatrix}$$

How do you access this sub-matrix $\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$

Matrix concatenation

- Matrix concatenation refers to merging of a row or column to a matrix

  - Concatenation of a row to a matrix is done using **rbind()**

  - Concatenation of a column to a matrix is done using **cbind()**

  - Consistency of the dimensions between the matrix and the vector should be checked before concatenation

Matrix concatenation – rbind()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 10 & 11 & 12 \end{bmatrix}$$

Use rbind() to append B row vector to the rows of A

$$C = \begin{bmatrix} A \\ B \end{bmatrix}$$

```
> C = rbind(A,B)
>
> C
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
[4,]   10   11   12
>
```

Matrix concatenation – cbind()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

Use cbind() to append B column vector to the columns of A

$$C = \begin{bmatrix} A & B \end{bmatrix}$$

```
> C = cbind(A,B)
>
> C
     [,1] [,2] [,3] [,4]
[1,]    1    2    3   10
[2,]    4    5    6   11
[3,]    7    8    9   12
>
```

Dimension inconsistency –cbind()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 10 & 11 & 12 \end{bmatrix}$$

Can these two matrices be merged to give

$$C = \begin{bmatrix} A & B \end{bmatrix}$$

```
> D = cbind(A,B)
Error in cbind(A, B) : number of rows of matrices must match (see arg 2)
```

Fixing the dimension inconsistency

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

$$C = \begin{bmatrix} A & B \end{bmatrix}$$

```
> C = cbind(A,B)
>
> C
     [,1] [,2] [,3] [,4]
[1,]    1    2    3   10
[2,]    4    5    6   11
[3,]    7    8    9   12
>
```

Deleting a column

$$A = \begin{bmatrix} 1 & \boxed{2} & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

- Access the column that has to be deleted and then put a '-' sign in front of it
  - For example: A=A[,-2]
  - This will fetch all the columns

    except the 2nd column

```
> A[,-2]
     [,1] [,2]
[1,]    1    3
[2,]    4    6
[3,]    7    9
```

Deleting a row

$$A = \begin{bmatrix} 1 & 2 & 3 \\ \boxed{4 \quad 5 \quad 6} \\ 7 & 8 & 9 \end{bmatrix}$$

- Access the row that has to be deleted and then put a '-' sign in front of it
  - For example: A=A[-2,]
  - This will fetch all the rows except the 2nd row

```
> A[-2,]
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    7    8    9
>
```

Matrix algebra

- **Addition/subtraction**
- **Multiplication**
- **Matrix Operations in R**
- **Matrix Division**

Matrix addition/subtraction & multiplication

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 9 & 1 \end{bmatrix}_{3\times 3}$$

$$B = \begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 1 \\ 5 & 1 & 2 \end{bmatrix}_{3\times 3}$$

```
Console ~/
> A+B
     [,1] [,2] [,3]
[1,]    4    3    6
[2,]    8    7    7
[3,]   13   10    3
> A-B
     [,1] [,2] [,3]
[1,]   -2    1    0
[2,]    0    3    5
[3,]    3    8   -1
> A%*%B # Regular matrix multiplication
     [,1] [,2] [,3]
[1,]   26    8   11
[2,]   62   20   29
[3,]   65   27   35
> A*B # element-wise multiplication
     [,1] [,2] [,3]
[1,]    3    2    9
[2,]   16   10    6
[3,]   40    9    2
>
```

Element-wise multiplication is based on multiplication between corresponding elements of two matrices.

Matrix division

WARNING :

The following operation is not inverse of a matrix but element wise division between matrices A & B.

$$A = \begin{bmatrix} 4 & 9 \\ 16 & 25 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\boxed{A/B = \frac{a_{ij}}{b_{ij}}}$$

Console ~/

```
> A= matrix(c(4,9,16,25),2,2)
> B= matrix(c(2,3,4,5),2,2)
> A
     [,1] [,2]
[1,]    4   16
[2,]    9   25
> B
     [,1] [,2]
[1,]    2    4
[2,]    3    5
> A/B
     [,1] [,2]
[1,]    2    4
[2,]    3    5
```